King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering

Information and Computer Science Department

ICS 253: Discrete Structures I

Summer Semester 2011-2012

Major Exam #1, Thursday June 28, 2012.

Name:

ID#:

**Instructions**:

1. This exam consists of **eight** pages, including this page, containing **four** questions.
2. You have to answer all **four** questions.
3. The exam is closed book and closed notes. No calculators or any helping aides are allowed. Make sure you turn off your mobile phone and keep it in your pocket if you have one.
4. The questions are **NOT equally weighed**. Some questions count for more points than others.
5. The maximum number of points for this exam is **100**.
6. You have exactly **90** minutes to finish the exam.
7. Make sure your answers are **readable**.
8. If there is no space on the front of the page, feel free to use the back of the page. Make sure you indicate this in order for me not to miss grading it.

|  |  |  |
| --- | --- | --- |
| Question Number | Maximum # of Points | Earned Points |
| 1 | **30** |  |
| 2 | **15** |  |
| 3 | **30** |  |
| 4 | **25** |  |
| **Total** | **100** |  |

|  |  |  |  |
| --- | --- | --- | --- |
|  | Addition |  | Modus Tollens |
|  | Simplification |  | Hypothetical syllogism |
|  | Conjunction |  | Disjunctive syllogism |
|  | Modus Ponens |  | Resolution |

1. **(30 points) Propositional Logic and Propositional Equivalences**
	1. (6 points) Construct a truth table for the compound proposition $\left(¬p\leftrightarrow ¬q\right)\rightarrow \left(q⊕r\right)$.
	2. (1 points) Determine whether the compound proposition in “a.” is a tautology, contradiction or contingency.
	3. (6 points) State the converse, contrapositive and inverse of the conditional statement: A positive integer is a prime only if it has no divisors other than 1 and itself.
	4. (10 points) Suppose that you meet three people, A, B, and C, on the island of knights and knaves described in Homework Assignment 1 (Knights always tell the truth and knaves always lie). What are A, B, and C if A says "I am a knave and B is a knight" and B says "Exactly one of the three of us is a knight"?
	5. (7 points) Show that  is a tautology without using a truth table.
2. **(15 points) Predicates and Quantifiers**
	1. (15 points) Let M (x,y) be "x has sent y an e-mail message" and T(x,y) be "x has telephoned y," where the domain consists of all students in your class. Use quantifiers to express each of these statements. (Assume that all e-mail messages that were sent are received, which is not the way things often work.) You may
		1. (2 points) Abdullah has never received an e-mail message from Rami.
		2. (2 points) Everyone in your class has either telephoned Muhammad or sent him an e-mail message.
		3. (3 points) There is someone in your class who has either sent an e-mail message or telephoned everyone else in your class.
		4. (3 points) There is a student in your class who has not received an e-mail message from anyone else in the class and who has not been called by any other student in the class.
		5. (5 points) There are two different students in your class who between them have sent an e-mail message to or telephoned everyone else in the class.
3. **(30 points) Rules of Inference and Methods of Proof**
	1. (5 points) What is wrong with this argument? Given the premise $∃xP(x)∧∃xQ(x)$, use simplification to obtain $∃xP(x)$; use existential instantiation to obtain *P*(*c*) for some element *c*; use simplification again to obtain $∃xQ(x)$; use existential instantiation to obtain *Q*(*c*) for some element *c*; use conjunction to conclude that $P(c)∧Q(c)$; and finally, use existential generalization to conclude that

$$∃x\left(P(x)∧Q(x)\right)$$

* 1. (10 points) Prove that if *x*3 is irrational, then *x* is irrational.
	2. (15 points) Show that these statements about the integer *x* are equivalent: (*i*) 3*x* + 2 is even, (*ii*) *x* + 5 is odd, (*iii*) *x*2 is even.
1. **(25 points) Sets, Set Operations and Functions**
	1. (3 points) Let *A* = {*a*, *b*, *c*}, B = {*x*, *y*}, and *C* = {0, 1}.

Find *A* × *C* × *B*.

* 1. (5 points) Find the truth set of each of these predicates where the domain is the set of integers.
		1. (2 points) *P*(x): "$x^{3}\geq 1$"
		2. (1 point) *Q*(x): "$x^{2}=2$"
		3. (2 points) *R*(x): "$x<x^{2}$"
	2. (5 points) For $A\_{i}=\{i,i+1,i+2,…\}$, where *i* is a positive integer,
		1. (2.5 points) Find 
		2. (2.5 points) Find 
	3. (3 points) Draw the Venn diagram for $\left(A∩B\right)∪\left(A∩C\right)$
	4. (6 points) Draw the graph of the function $\left⌊x\right⌋+\left⌈\frac{x}{2}\right⌉$ for .
	5. (3 points) Prove that if *x* is a real number, then ⎣− *x* ⎦ = − ⎡*x*⎤.